Astronomical Computing

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Sunrise and Sunset: A Challenge

"WHERE CAN I get the formulas to calculate the times of sunrise and sunset?" To this frequent query I must respond that the topic has never come up in the Astronomical Computing department. How can so many contributors in the last 10 years have overlooked such an obvious calculation?

No doubt many of us have a workable if clumsy routine, perhaps downloaded from a computer bulletin board or adapted from the euphoric early days of programmable calculators. My own program, presented here, duplicates to the minute the values listed in the annual *Astronomical Almanac* and even handles unusual conditions in the arctic regions. But I have a lingering hunch that a few simple lines of code could do the job almost as well, which leads to the challenge at the end of this article.

At a tranquil moment like this, when the Sun's disk *appears* to sit upon the horizon, it would in fact be entirely concealed if the Earth had no air. Acting to raise the disk into view, atmospheric refraction is one of the factors that should be taken into account when calculating the time of sunrise or sunset. Photograph by Pekka Parviainen of Turku, Finland.

SPOTLIGHTING THE PROBLEM

Sunrise takes place when the first gleam of sunlight appears on the eastern horizon, and sunset when the last gleam disappears from view in the west. By convention astronomers say that sunrise or sunset occurs when the Sun's center lies 50' below the horizon, allowing 16' for the radius of the solar disk and 34' for atmospheric refraction to lift the upper limb of the Sun to the horizon.

In the accompanying program, written in generic Basic for an observer at sea level, the computer first asks for the latitude, longitude, and time zone. Be sure to enter north latitudes positive, and west longitudes *negative*, in accord with current astronomical practice. The time zone is just the whole number of hours you routinely add for converting civil to Universal Time. Finally, enter the year, month, and day for which you want the rise and set times.

For example, Charlottesville, Virginia, is located at north latitude 38°.0, west longitude 78°.5, and keeps Eastern Daylight Time this summer. An observer there can enter "38.0, –78.5," and "4." On August 4th of this year (enter "1994, 8, 4"), the program reports that sunrise occurs at 6:19 and sunset at 20:20 (that is, 8:20 p.m.) EDT. At these times the solar azimuth is 67°.2 and 292°.6, respectively, where 90° would be due east and 270° due west.

This program is patterned on a similar one published earlier for moonrise and moonset (*S&T:* July 1989, page 78). It calculates the Sun's right ascension and declination to an accuracy of 0°.01 at the start and end of each day using the routine in lines 910–1150. Then, with the help of the local sidereal time found in lines 410–470, it checks to see if the Sun crosses the horizon during each successive hour of the day and which direction it is heading (lines 490–800). In this way the program locates every sunset or sunrise, even those occurring in reverse order or missing, as sometimes occurs in the polar regions.

The northernmost town in the United States is Barrow, Alaska, at latitude 71°.3 north and longitude 156°.8 west. For this remote locale the program reports a sunset at 1:47 p.m. Alaska Standard Time on November 18, 1994, but no sunrise the next day — the long winter night has begun! Barrow's sunrises resume January 23, 1995, at 1:09 p.m.

GREATER EFFICIENCY?

My program uses a standard approach, treating the Sun the same as any star or planet and plodding through the calculation via the intermediaries of right ascension and sidereal time. But the Sun really is unique among celestial objects — our entire calendar and civil time system are tailored to the Sun's daily and yearly cycles. Otherwise, sundials would not work.

In a supplement to the 1946 *American Ephemeris,* the U. S. Naval Observatory took advantage of the Sun's nearly exact repetition from year to year. Titled *Tables of Sunrise, Sunset, and Twilight,* this 196-page book lists times for every day of the year and each degree of latitude from 0° to 75°. The error of a looked-up time, adjusted for an observer's local time zone, never exceeds 3 minutes in the contiguous United States throughout the last half of the 20th century.

To achieve this accuracy the data were calculated for 1966 (midway between two leap years) and longitude 90° west. For further refinement or a place elsewhere in the world, the text at the back of the book explains how to interpolate for an exact latitude and apply very small corrections for the specific year, longitude, and elevation above sea level.

This work was compiled by Wallace J. Eckert and G. M. Clemence at the dawn of the computer age, using the business machinery of the day. They give some idea of what was involved:

> The first step in the actual computation consisted of constructing a table on punched cards giving tan δ, sec δ, and the equation of time, with argument the month, day, hour, and minute of local civil time. The table contains more than 17,000 cards. [Several more card decks were prepared,] then run twice through the multiplying punch. . . . All the cards were sorted together in such a way that when they were tabulated, it was possible to summary punch in one operation the local civil times of sunrise and sunset. . . . The summary punch cards were then sorted so as to off-gang punch the time of sunset for the following day. . . . From these cards the final cards were reproduced in the form needed to control the card-operated typewriter.

Eckert and Clemence don't say how long it took to prepare their massive tables, but in checking my program I redid their entire calculation on my 486 50-MHz machine — in a breezy 92 seconds!

THE CHALLENGE

Despite being saddled with primitive equipment, Eckert and Clemence pointed the way to how we might write a very compact computer routine for general purposes. The equation of time, familiar to sundial users, lets us avoid specifying the year.

So just how short and sweet can a sunrise-sunset program be and still give results accurate to within a few minutes? Send your own computer routine to this department, writing "sunrise" on the envelope. I'll select the best examples for publication in a future issue.

R. W. S.

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10 '        Sunrise-Sunset
20 GOSUB 300
30 INPUT "Lat, Long (deg)";B5,L5
40 INPUT "Time zone (hrs)";H
50 L5=L5/360: Z0=H/24
60 GOSUB 1170: T=(J-2451545)+F
70 TT=T/36525+1: ' TT = centuries
80 '                    from 1900.0
90 GOSUB 410: T=T+Z0
100 '
110 '    Get Sun's Position
120 GOSUB 910: A(1)=A5: D(1)=D5
130 T=T+1
140 GOSUB 910: A(2)=A5: D(2)=D5
150 IF A(2)<A(1) THEN A(2)=A(2)+P2
160 Z1=DR*90.833: ' Zenith dist.
170 S=SIN(B5*DR): C=COS(B5*DR)
180 Z=COS(Z1): M8=0: W8=0: PRINT
190 A0=A(1): D0=D(1)
200 DA=A(2)-A(1): DD=D(2)-D(1)
210 FOR C0=0 TO 23
220 P=(C0+1)/24
230 A2=A(1)+P*DA: D2=D(1)+P*DD
240 GOSUB 490
250 A0=A2: D0=D2: V0=V2
260 NEXT
270 GOSUB 820: '   Special msg?
280 END
290 '
300 '           Constants
310 DIM A(2),D(2)
320 P1=3.14159265: P2=2*P1
330 DR=P1/180: K1=15*DR*1.0027379
340 S$="Sunset at  "
350 R$="Sunrise at "
360 M1$="No sunrise this date"
370 M2$="No sunset this date"
380 M3$="Sun down all day"
390 M4$="Sun up all day"
400 RETURN
410 '       LST at 0h zone time
420 T0=T/36525
430 S=24110.5+8640184.813*T0
440 S=S+86636.6*Z0+86400*L5
450 S=S/86400: S=S-INT(S)
460 T0=S*360*DR
470 RETURN
480 '
490 '   Test an hour for an event
500 L0=T0+C0*K1: L2=L0+K1
510 H0=L0-A0: H2=L2-A2
520 H1=(H2+H0)/2: '   Hour angle,
530 D1=(D2+D0)/2: '   declination,
540 '                  at half hour
550 IF C0>0 THEN 570
560 V0=S*SIN(D0)+C*COS(D0)*COS(H0)-Z
570 V2=S*SIN(D2)+C*COS(D2)*COS(H2)-Z
580 IF SGN(V0)=SGN(V2) THEN 800
590 V1=S*SIN(D1)+C*COS(D1)*COS(H1)-Z
600 A=2*V2-4*V1+2*V0: B=4*V1-3*V0-V2
610 D=B*B-4*A*V0: IF D<0 THEN 800
620 D=SQR(D)
630 IF V0<0 AND V2>0 THEN PRINT R$;
640 IF V0<0 AND V2>0 THEN M8=1
650 IF V0>0 AND V2<0 THEN PRINT S$;
660 IF V0>0 AND V2<0 THEN W8=1
670 E=(-B+D)/(2*A)
680 IF E>1 OR E<0 THEN E=(-B-D)/(2*A)
690 T3=C0+E+1/120: ' Round off
700 H3=INT(T3): M3=INT((T3-H3)*60)
710 PRINT USING "##:##";H3;M3;
720 H7=H0+E*(H2-H0)
730 N7=-COS(D1)*SIN(H7)
740 D7=C*SIN(D1)-S*COS(D1)*COS(H7)
750 AZ=ATN(N7/D7)/DR
760 IF D7<0 THEN AZ=AZ+180
770 IF AZ<0 THEN AZ=AZ+360
780 IF AZ>360 THEN AZ=AZ-360
790 PRINT USING ", azimuth ###.#";AZ
800 RETURN
820 '    Special-message routine
830 IF M8=0 AND W8=0 THEN 870
840 IF M8=0 THEN PRINT M1$
850 IF W8=0 THEN PRINT M2$
860 GOTO 890
870 IF V2<0 THEN PRINT M3$
880 IF V2>0 THEN PRINT M4$
890 RETURN
910 '    Fundamental arguments
920 '        (Van Flandern &
930 '        Pulkkinen, 1979)
940 L=.779072+.00273790931*T
950 G=.993126+.0027377785*T
960 L=L-INT(L): G=G-INT(G)
970 L=L*P2: G=G*P2
980 V=.39785*SIN(L)
990 V=V-.01000*SIN(L-G)
1000 V=V+.00333*SIN(L+G)
1010 V=V-.00021*TT*SIN(L)
1020 U=1-.03349*COS(G)
1030 U=U-.00014*COS(2*L)
1040 U=U+.00008*COS(L)
1050 W=-.00010-.04129*SIN(2*L)
1060 W=W+.03211*SIN(G)
1070 W=W+.00104*SIN(2*L-G)
1080 W=W-.00035*SIN(2*L+G)
1090 W=W-.00008*TT*SIN(G)
1110 '  Compute Sun's RA and Dec
1120 S=W/SQR(U-V*V)
1130 A5=L+ATN(S/SQR(1-S*S))
1140 S=V/SQR(U):D5=ATN(S/SQR(1-S*S))
1150 RETURN
1170 '        Calendar --> JD
1180 INPUT "Year, Month, Day";Y,M,D
1190 G=1: IF Y<1583 THEN G=0
1200 D1=INT(D): F=D-D1-.5
1210 J=-INT(7*(INT((M+9)/12)+Y)/4)
1220 IF G=0 THEN 1260
1230 S=SGN(M-9): A=ABS(M-9)
1240 J3=INT(Y+S*INT(A/7))
1250 J3=-INT((INT(J3/100)+1)*3/4)
1260 J=J+INT(275*M/9)+D1+G*J3
1270 J=J+1721027+2*G+367*Y
1280 IF F>=0 THEN 1300
1290 F=F+1: J=J-1
1300 RETURN
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