

Statistics: 1.2 Unpaired t-tests

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1 Introduction

An unpaired t-test is used to compare two population means. The following notation will be used throughout this leaflet:

Group	Sample size	Sample mean	Sample standard deviation
1	n_1	\bar{x}_1	s_1
2	n_2	\bar{x}_2	s_2

2 Procedure for carrying out an unpaired t-test

To test the null hypothesis that the two population means, μ_1 and μ_2 , are equal:

1. Calculate the difference between the two sample means, $\bar{x}_1 - \bar{x}_2$.
2. Calculate the **pooled standard deviation**: $s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$
3. Calculate the standard error of the difference between the means:

$$SE(\bar{x}_1 - \bar{x}_2) = s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

4. Calculate the T-statistic, which is given by $T = \frac{\bar{x}_1 - \bar{x}_2}{SE(\bar{x}_1 - \bar{x}_2)}$. Under the null hypothesis, this statistic follows a t-distribution with $n_1 + n_2 - 2$ degrees of freedom.
5. Use tables of the t-distribution to compare your value for T to the $t_{n_1 + n_2 - 2}$ distribution. This will give the p-value for the unpaired t-test.

NOTE:

For the unpaired t-test to be valid the two samples should be roughly normally distributed and should have approximately equal variances. If the variances are obviously unequal we must use:

$$SE(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Then: $\frac{\bar{x}_1 - \bar{x}_2}{SE(\bar{x}_1 - \bar{x}_2)} \sim N(0, 1)$ if n_1 and n_2 are reasonably large.

Else: $\frac{\bar{x}_1 - \bar{x}_2}{SE(\bar{x}_1 - \bar{x}_2)} \sim t_{n'}$, where $n' = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{(n_1 - 1)} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{(n_2 - 1)}}$ rounded down to the nearest integer.

Example:

(Data taken from Moore and McCabe – Introduction to the Practice of Statistics)

A U.S. magazine, Consumer Reports, carried out a survey of the calorie and sodium content of a number of different brands of hotdog. There were three types of hotdog: beef, 'meat' (mainly pork and beef but can contain up to 15% poultry) and poultry. The results below are the calorie content of the different brands of beef and poultry hotdogs.

Beef hotdogs:

186, 181, 176, 149, 184, 190, 158, 139, 175, 148, 152, 111, 141, 153, 190, 157, 131, 149, 135, 132

Poultry hotdogs:

129, 132, 102, 106, 94, 102, 87, 99, 170, 113, 135, 142, 86, 143, 152, 146, 144

Before carrying out a t-test you should check whether the two samples are roughly normally distributed. This can be done by looking at histograms of the data. In this case there are no outliers and the data look reasonably close to a normal distribution; the t-test is therefore appropriate. So, first we need to calculate the sample mean and standard deviation in each group:

Group	Sample size	Sample mean	Sample standard deviation
Beef	20	156.85	22.64
Poultry	17	122.47	25.48

So, we have $\bar{x}_1 - \bar{x}_2 = 156.85 - 122.47 = 34.38$

The standard deviations are approximately equal, so we can calculate the pooled standard deviation:

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{(19)22.64^2 + (16)25.48^2}{35}} = 23.98$$

We can now calculate $SE(\bar{x}_1 - \bar{x}_2)$:

$$s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 23.98 \sqrt{\frac{1}{20} + \frac{1}{17}} = 7.91$$

And now the value for T:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{SE(\bar{x}_1 - \bar{x}_2)} = \frac{34.38}{7.91}$$

If we look this up in tables of the t-distribution with 35 degrees of freedom, we find $p < 0.001$. Therefore, there is strong evidence that the calorie content of poultry hotdogs is lower than the calorie content of beef hotdogs.

3 Confidence interval for the true mean difference

It would also be useful to calculate a confidence interval for the difference in means to tell us within what limits the true difference is likely to lie. A 95% confidence interval for the true difference between the means is given by:

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \times s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \quad \text{or, equivalently} \quad (\bar{x}_1 - \bar{x}_2) \pm t^* \times SE(\bar{x}_1 - \bar{x}_2)$$

where t^* is the 2.5% point of the t-distribution on $n_1 + n_2 - 2$ degrees of freedom.

Using our example:

We have a mean difference of 34.38 calories. The 2.5% point of the t-distribution with 35 degrees of freedom is 2.03. The 95% confidence interval for the true mean difference is therefore:

$$34.38 \pm (2.03 \times 7.91) = 34.38 \pm 16.06 = (18.3, 50.4)$$

We can be 95% sure that the mean calorie content of beef hotdogs is somewhere between 18 and 50 calories higher than the mean calorie content of poultry hotdogs.

4 Carrying out an unpaired t-test in SPSS

- **Analyze**
- **Compare Means**
- **Independent-Samples T-Test**
- Choose your outcome variable (in our case calories) as the **Test Variable**
- Choose the variable that defines the groups as the **Grouping Variable** then click on **Define Groups**. You will need to enter the two codes which identify your two groups then click on **Continue** (in our case, the code 1 was used for Beef and the code 2 for Poultry, so we would enter 1 in **Group 1** and 2 in **Group 2**). Now click **OK**.

The output will look something like this:

Group Statistics					
	GROUP	N	Mean	Std. Deviation	Std. Error Mean
CALORIES	Beef	20	156.85	22.642	5.063
	Poultry	17	122.47	25.483	6.181

Independent Samples Test									
	Levene's test for Equal Variances		t-test for Equality of Means						
	F	Sig.	t	df	Sig. (2-tailed)	Mean Diff.	SE Diff.	95% C.I.	
								Lower	Upper
Eq. variances assumed	1.046	0.314	4.346	35	.000	34.38	7.911	18.318	50.441
Eq. variances not assumed			4.303	32.394	.000	34.38	7.990	18.113	50.646

The relevant results in the second table are given in the row where equal variances are assumed. You are interested in t, the p-value (Sig.(2-tailed)), the mean difference and the 95% confidence interval.